647510 **MSC1MatC102x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
Oct/Nov. -2019 - [NEW COURSE]
Real Analysis (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 Answer following short questions: (Any SEVEN) (14)

(i) Write all possible algebra of sets for a set $X = \{a, b, c\}$.

(ii) Define σ −algebra of sets.

(iii) Show that an outer measure of a singleton set is zero.

(iv) Show that a null set is measurable set.

(v) Discuss Riemann Integral.

(vi) Define point-wise convergence.

(vii) Define measurable function.

(viii) Explain Function of Bounded Variation.

(ix) State Fatou's lemma.

(x) Define absolute continuity

Q.2 Attempt any TWO. (14)

(a) If f is measurable function and $f = g\ a.e.$ then show that g is measurable function.

(b) Define Characteristic function and prove that a set A is measurable if and only if its characteristic function χ_A is measurable.

(c) Show the existence of non measurable set.

Q.3 Attempt the following. (14)

(a) State and prove Bounded convergence theorem.

(b) Let f and g be integrable functions over E then prove that the function cf is integrable over E and $\int_E (cf) = c \int_E f$.

OR

Q.3 Attempt the following. (14)

(a) Let f and g are bounded measurable function defined on a set E of finite measure then prove that $\int_E (af + bg) = a \int_E f + b \int_E g$.

(b) State and prove monotone convergence theorem.

Q.4 Attempt any TWO. (14)

(a) Let f be an increasing real – valued function on the interval $[a, b]$. Then show that f is differentiable everywhere and f' is measurable.

(b) If f is of bounded variation on $[a, b]$ then prove in usual notations $T = P + N$.

(c) If f is absolute continuous on $[a, b]$ then show that it is of bounded variation on $[a, b]$.

Q.5 Attempt any TWO. (14)

(a) Prove that a function F is an indefinite integral if and only if it is absolute continuous.

(b) State and prove Minkowski's inequality.

(c) State and prove Holder's inequality.

(d) If a normed linear space X is complete then prove that every absolutely summable series in X is summable.

****